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Quantifying mis-modelling effects in the GNSS yaw-attitude and phase wind-up

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ABSTRACT

The yaw-attitude modelling of GNSS satellites is a critical objective underlying the phase wind-up (PWU) correction and orbit determination. The attitude mis-modelling during the eclipse seasons may lead to a significant impact in the Precise Point Positioning (PPP), applications with demanding availability requirements. The current study focuses on the numerical investigation of the GNSS yaw-attitude model as this has been implemented into our Analysis Centre Software (ACS) Orbit Determination and Positioning (ODP) module. We focus on periods during the eclipse and their impact on the PWU by considering tabulated GNSS precise orbit data. The present analysis aims at quantifying underlying issues in precise applications such as the impact of the yaw-angle mis-modelling to the PWU effect. The yaw-attitude analysis has been performed for daily and annual time periods. The modelling of the yaw-angle variations over annual periods, as a function of the β angle during a GNSS draconic year forms a useful prediction scheme of the GNSS eclipse seasons and the expected impact in the user's observations. The current approach has been applied for satellites included in the GPS Blocks IIA, IIR, IIF and the GLONASS-M series.

KEYWORDS: Yaw attitude models, GNSS Eclipses, Phase wind-up, Precise Orbits

1. INTRODUCTION

Satellite motion may be comprised by the orbital motion of the satellite centre of mass and the orientation of the satellite body w.r.t. the inertial frame. In the case of the Global Navigation Satellite Systems (GNSS) the orientation or attitude of the spacecraft body is critical and therefore, the attitude is controlled through on-board systems. The attitude control on the

GNSS satellites is dictated by two requirements or constraints, namely the transmitting antenna always points toward the earth and that the solar panel axis is perpendicular to the Sun direction. These requirements necessitate that the satellites constantly rotate along the antenna axis which point to the Earth. This attitude behaviour, characterized as nominal attitude, is achieved by performing constant changes in the yaw angle i.e. the rotation angle around the satellites z-axis which is pointing towards the centre of the earth. The reference direction is the yaw origin, a unit vector to form an orthogonal basis for the orbit plane and is in the general direction of the satellite velocity vector (Bar-Sever 1996).

However the nominal attitude, as defined according to the two aforementioned conditions, is not valid during the Sun-Earth eclipses seasons of the GNSS satellites and therefore, mis-modelling effects may arise (Bar-Sever et al. 1995, Bar-Sever 1996, Kouba 2009a, Dilssner et al. 2011). The attitude mis-modelling during the eclipse seasons may lead to a significant impact in the Precise Point Positioning (PPP) solutions (Lu et al. 2015, Zhang et al. 2010) and the Precise Orbit Determination (POD) accuracy (Rodriguez-Solano et al. 2013, Guo et al. 2016).

The current study aims at predicting underlying issues in the estimation of the phase-wind up (PWU) correction due to the yaw-angle mis-modelling. We focus on the investigation of the eclipse periods by considering various tabulated orbit data during different time periods. The numerical investigation has been applied for satellites included in the GPS Blocks IIA, IIR, IIF and the GLONASS-M series. The yaw-angle and PWU analysis has been performed during daily and annual time periods as well. In particular, by extending the yaw-angle modelling scheme into annual periods, as a function of the β angle (the elevation of the sun above the satellite's orbital plane) during a full GNSS draconic year, we can accurately predict the mis-modelling of attitude regimes during GNSS eclipse seasons, and the expected impact on the PWU. On the other end, PPP users can now be provided with a yaw-angle and yaw-rate model for consistent computation of the PWU effect. The effect is quantified during the eclipse seasons and the corresponding impact in the carrier phase observations is presented as well. The investigation depicts that depending from the constellation type, an important number of observations during eclipse seasons, can produce a significant decrease in PPP accuracy, and notably inaccurate ambiguity fixing.

2. Yaw-attitude models

In the present analysis, the GNSS body-fixed reference frame definition is followed according to the International GNSS Service (IGS) conventions (Montenbruck et al. 2015) which are based on the spacecraft body frame of the GPS Block II/IIA satellites. This definition is also compatible with the GPS Block IIF satellites while in the case of the GPS Block IIR the spacecraft frame is designed with a reverse direction in the X and Y axes of the body-fixed frame.

The unit vector of the X axis of the body-fixed frame may be computed during nominal yaw-attitude conditions as follows

$$e_x^{BF} = -\left(e^{sun} \times e^{sat}\right) \times e_x^{sat} \tag{1}$$

where e^{sun}, e^{sat} are the unit vectors of the Sun and the satellite in the inertial frame respectively.

The β angle between the Sun position vector and the satellite orbital plane is defined according to the following equation

$$\beta = \cos^{-1}\left[\left(e_{\upsilon}^{sat} \times e^{sat}\right) \cdot e_{x}^{sat}\right] - \pi/2 \tag{2}$$

where e_{υ}^{sat} is the satellite's velocity unit vector in the inertial frame.

The nominal yaw angle may then be computed by the following equation (Bar-Sever 1996)

$$\psi = ATAN2(-\tan\beta, \sin\mu) \tag{3}$$

while in the case of the GPS Block IIR, the equation is applied as follows due to the reverse sign of the X axis of the body-fixed frame

$$\psi = ATAN2(\tan\beta, -\sin\mu) \tag{4}$$

where ψ is the yaw angle and μ is the orbital angle between satellite position vector and the orbit midnight i.e. the orbit point which is furthest from the Sun. The $ATAN2$ follows the conventions of the FORTRAN function.

During the Sun-Earth eclipse seasons, the nominal attitude, as computed by the above formulas, is not valid when the β angle approaches low values (Kouba 2009a). The eclipsing attitude may be divided in three periods i.e. the noon and midnight turns, the shadow crossing and the post-shadow recovery period which occurs to GPS IIA satellites (Bar-Sever 1996, Kouba 2009a).

The yaw angle during noon or midnight turns and shadow crossing may be computed by the following equations

$$\psi(t) = ATAN2(-\tan\beta, \sin\mu) + SIGN(R, \beta)(t - t_s), \tag{5}$$

$$\psi(t) = ATAN2(\tan\beta, -\sin\mu) + SIGN(R, \beta)(t - t_s) \quad \text{for GPS Block IIR only,} \tag{6}$$

where t is the computation epoch, t_s is the start time of the eclipsing turn and $SIGN$ refers to the sign conventions of the relevant Fortran function. The R denotes the nominal hardware yaw rates.

In the case of a post-shadow recovery, which is suitable for the GPS Block IIA satellites, the following equation is applied

$$\psi(t) = \psi(t_e) + SIGN(R, \Delta\psi)(t - t_e) \tag{7}$$

where t_e is the end time of the shadow crossing period and $\Delta\psi$ is the difference between $\psi(t_e)$ and the nominal attitude at the computation epoch t.

3. Phase wind-up (PWU) effect

The satellite attitude variation and the corresponding changes in the spacecraft body-fixed frame produce rotations to the GNSS satellite antenna. The antenna rotations have an impact to the observations and in particular, the phase carrier signals. This is well known as the phase wind-up (PWU) effect (Wu et al. 1993). Thus, the PWU correction should be applied in the precise GNSS applications (Kouba 2009b) and it is given as follows (Wu et al. 1993)

$$\Delta\varphi = sign(\zeta)\cos^{-1}\left(\frac{\vec{D}'\cdot\vec{D}}{\left|\vec{D}'\right|\left|\vec{D}\right|}\right) \tag{8}$$

where $\zeta = e^{loc}\left(\vec{D}'\times\vec{D}\right)$, e^{loc} is the satellite to receiver line-of-sight (loc) unit vector.

The $\vec{D}',\vec{D}$ are the effective dipole vectors of the satellite and receiver computed based on the unit vectors of the satellite's body-fixed b-frame (e_x^b, e_y^b, e_z^b) and the receiver's local level or local geodetic ll-frame East, North, Up (ENU) as follows

$$\vec{D}' = e_x^b - e^{loc}\left(e^{loc}\cdot e_x^b\right) - e^{loc}\times e_y^b\,, \tag{9}$$

$$\vec{D} = e_N^{ll} - e^{loc}\left(e^{loc}\cdot e_N^{ll}\right) - e^{loc}\times e_E^{ll}\,. \tag{10}$$

According to the above equations, it is well understood that the PWU correction is directly affected by the changes in the b-frame orientation due to the yaw-attitude mis-modelling during eclipse seasons.

4. Results

In the present analysis, the yaw-attitude modelling is followed as discussed in Section 2. The attitude models applied here have been introduced by Kouba (2009a), Dilssner (2010) and Dilssner et al. (2011) for the GPS Blocks IIA/IIR, GPS Block IIF and GLONASS-M satellites respectively. In particular, our source code applies the models and data processing as summarized in Table 1. The computations have been performed over daily and annual time periods. In the following, we refer to the differences between the nominal and the modelled attitude. The nominal attitude is defined as the "expected" attitude and it is computed based on the equations 3 and 4. The modelled attitude refers to the attitude during eclipse seasons as described by equations 5 through 7 and the application of the aforementioned models (Kouba 2009a, Dilssner 2010, Dilssner et al. 2011).

4.1 Daily orbit arcs

In the case of daily periods, the nominal and modelled attitude has been computed over orbit arcs obtained from the processing of precise orbit data. The precise orbit data applied have been obtained from the IGS final & rapid sp3 products. The available orbits provide the GNSS satellite's position vector with data interval equal to 15 min. An interpolation scheme has been applied in order to obtain the position and velocity vectors at a dense rate of 30 sec. The orbit interpolation has been implemented based on Lagrange polynomials with degree set equal to 11.

Yaw Attitude models	GPS (Kouba 2009, Dilssner 2010) GLONASS-M (Dilssner et al. 2011)
Orbit arc length	1. Daily arcs 2. Annual periods (GPS draconic year)
GNSS orbits	1. IGS final/rapid products, RSO data by GFZ Orbit interpolation: Lagrange polynomials 11th degree 2. Keplerian orbits
Planetary Ephemeris	DE430 (Folkner et al. 2014)
Earth Orientation	IERS Conventions 2010 (Petit and Luzum 2010)
EOP data	IERS 08 C04 series (Bizouard and Gambis 2009)
Precession-Nutation model	IAU 2000A (Mathews et al. 2002)
EOP tidal variations	IERS Conventions 2010 (Petit and Luzum 2010)

Table 1. Summary of features and data applied for the yaw-attitude modelling.

In order to validate the performance of the implemented orbit interpolation, a numerical comparison has been performed with the GPS Rapid Science Orbit (RSO) data provided by GeoForschungsZentrum (GFZ). The RSO data provide position and velocity vectors for the GPS satellites with data intervals equal to 30 seconds. The obit comparison between the RSO orbits and the interpolated orbits based on IGS rapid data has led to differences up to the level of 20 cm and 25 cm/sec for the position and velocity vectors respectively. The impact on the yaw-angle and the PWU effect was found to be negligible.

Equations 1 through 7 refer to the inertial reference frame, while the GNSS orbits are provided in the terrestrial reference frame. Therefore, a transformation from the terrestrial to the inertial frame is required. For this purpose, the rigorous transformation implemented in the ACS Time and Reference System (TRS) module is applied according to the International Earth Orientation and Reference Systems Service (IERS) Conventions (Petit and Luzum 2010). The required Earth Orientation Parameters (EOP) data were obtained from the 08 C04 solution (Bizouard and Gambis 2009) as provided by the IERS Earth Orientation Center (EOC). Tidal variations in the EOP data were taken into account based on the IERS Conventions 2010 (Petit and Luzum 2010).

The computation of the sun's elevation above the orbital plane, β angle, requires the sun's position vector as shown in Equation 2. This is achieved through the processing of the precise planetary/lunar ephemeris DE430 (Folkner at al. 2014).

As mentioned above, the models and data used in the present computations are summarized in Table 1. Based on this modelling, the results for the nominal and modelled yaw-angle variations during daily orbit arcs are illustrated in Figure 1. The computations were performed for GPS Blocks IIA, IIR, IIF and GLONASS-M satellites.

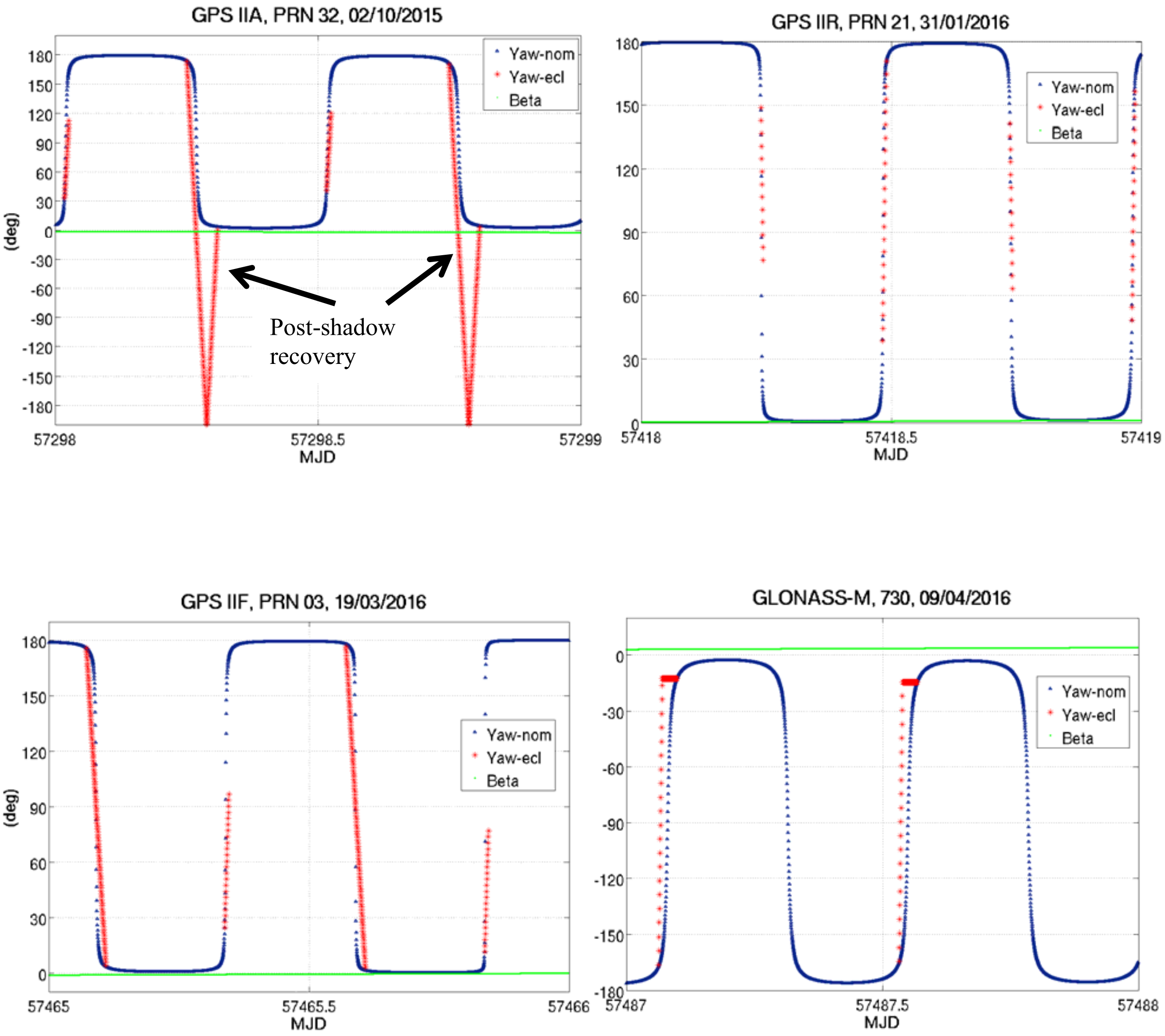


Figure 1. Nominal vs modelled eclipsing yaw-angle variations over a daily arc for GPS Blocks IIA, IIR, IIF and GLONASS-M series. The x-axis refers to the Modified Julian Days (MJD).

As it is easily identified, wide variations in the eclipsing yaw-angle are obtained for the GPS Block IIA satellite. In addition, a post-shadow recovery period has been applied only for the GPS IIA satellites (Figure 1 top-left). Since there is a large uncertainty in the attitude modelling during this time period (Kouba 2009a), it is a common approach to delete these observations as suggested by Bar-Sever (1996). The post-shadow arcs have a length of 30 min approximately. However, the current algorithm includes these periods, and the exit recovery is widely applied in the yaw-attitude modelling based on equation 7. Based on that, a user can still receive yaw-angle estimates during a post-shadow manoeuvre of the satellite and increase the availability of GNSS satellite data in view.

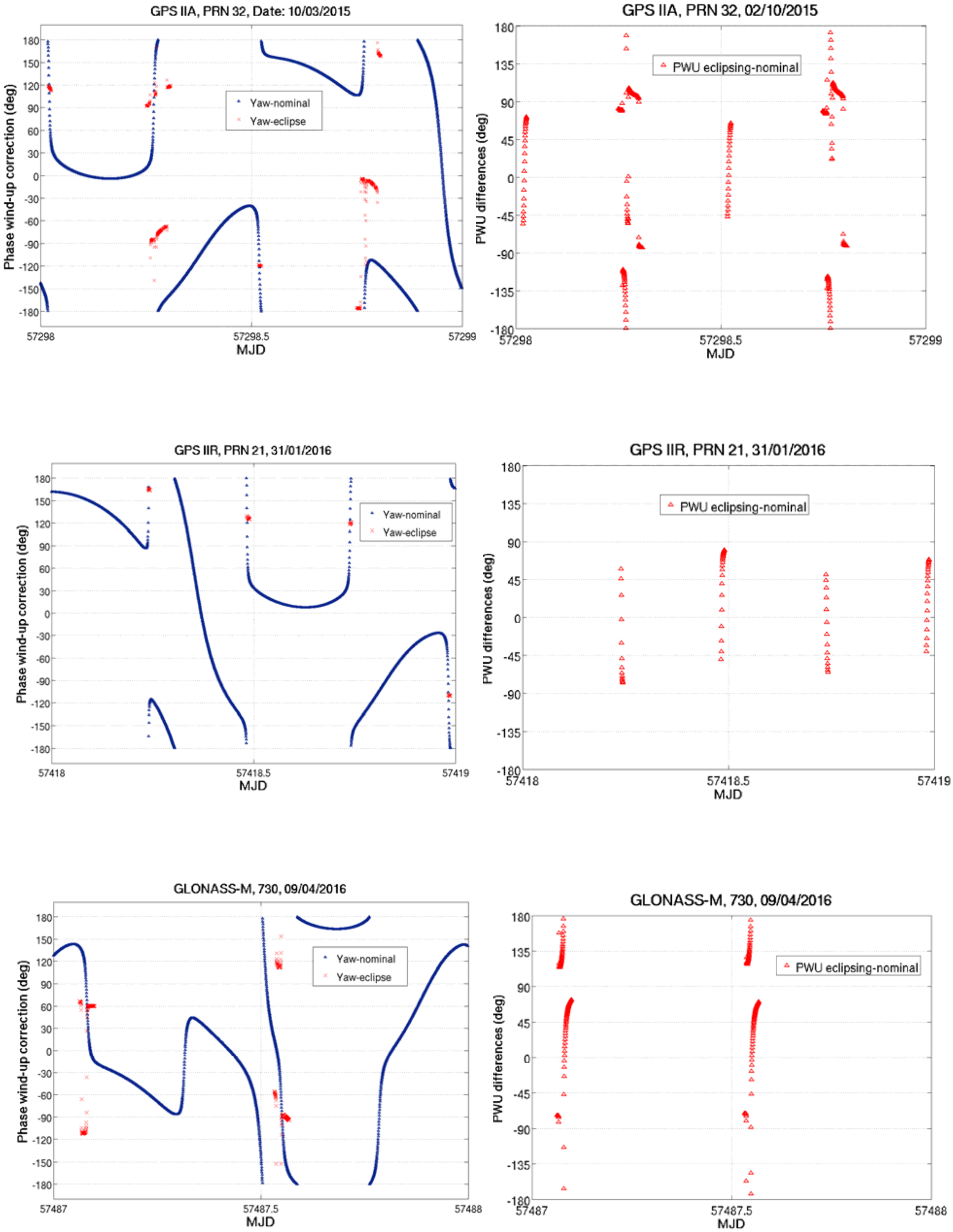


Figure 2. (Left) PWU correction variations over a daily period based on the nominal (blue) and modelled yaw-attitude (red). (Right) PWU correction differences over the same period. The PWU corrections have been computed for the IGS station Stromlo located in Australia.

The GPS IIR satellite follows closely the nominal attitude (Figure 1 top-right), which in general agrees with previous investigations (Kouba 2009a, Dilssner at al. 2011).

The PWU correction has been computed according to equations 8 through 10. The satellite body-fixed frame has been defined considering the computed nominal and modelled attitude during the eclipse seasons. The daily variations of the PWU correction for the IGS station Stromlo station located in Canberra, Australia are shown in Figure 2. The differences in the PWU correction between the nominal and modelled yaw-attitude during the eclipse season are also illustrated in the same figure (left). Special emphasis is given in those differences that exceed the range of ±45 degrees (quarter of a carrier phase cycle). This threshold has been considered as critical since it may produce an impact of about 3 cm in the phase observations depending on the frequency band. It is pronounced that in the case of the GPS IIR satellite the variations are kept within the limits of the considered threshold (middle of Figure 2). In the case of GLONASS-M and GPS IIA, the translated PWU effect is stronger and may reach values up to 180^0 (half a cycle) while a significant number of values is greater than 100^0.

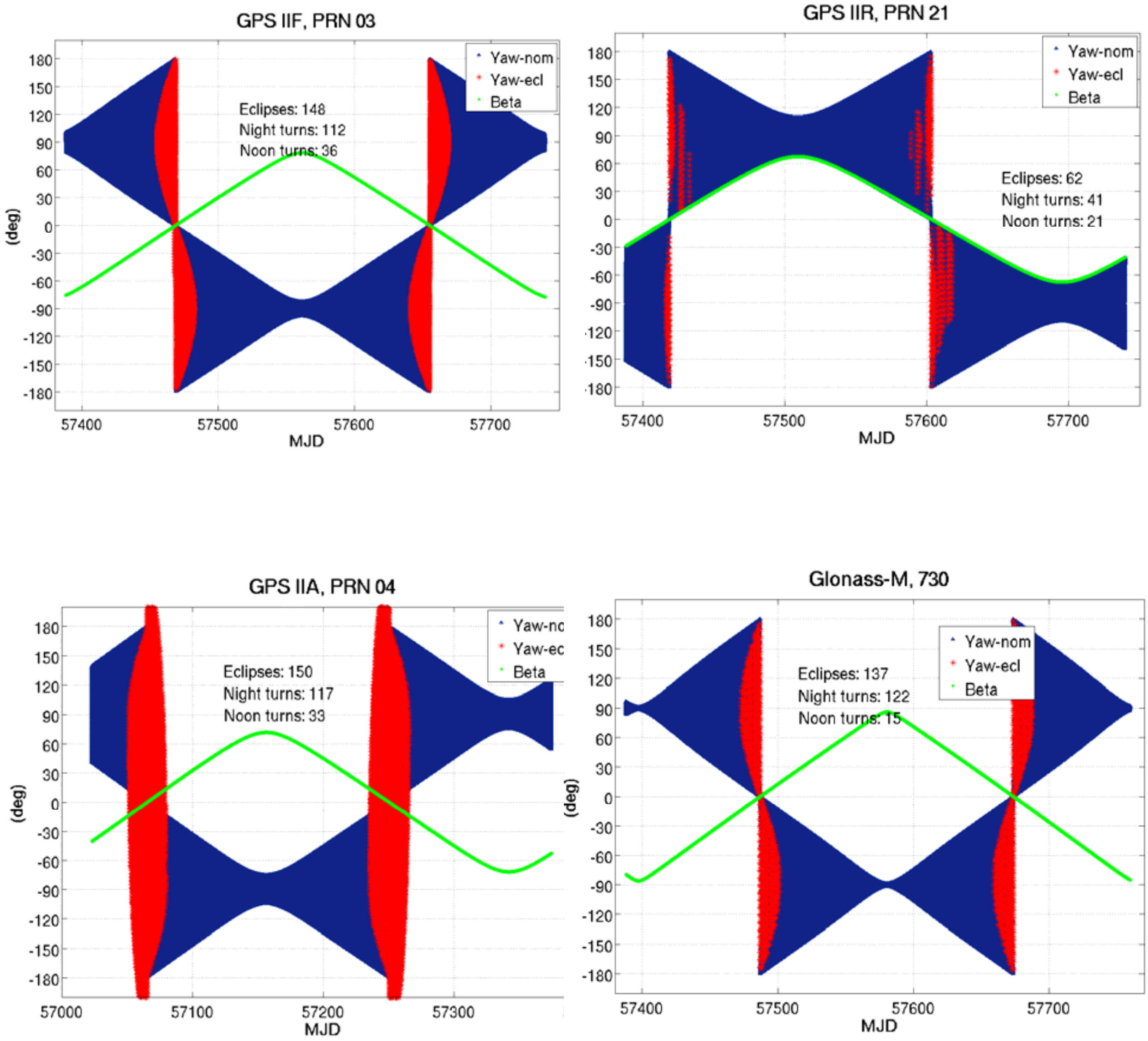
4.2 Annual periods

The GNSS eclipse seasons are depended from the Sun’s position with respect to the satellite orbital plane i.e. the β angle, which enters the equations for the yaw angle. In order to monitor the yaw-angle variations as a function of the β angle, the computing scenario has been implemented for long orbit arcs covering annual periods. Such an approach may capture the overall changes in the β angle within a full draconitic year (Schmid et al. 2007) and its impact to the yaw-attitude behaviour.

In the case of the annual periods, the GNSS orbit arcs have been approximated by Keplerian orbits. The basic formulas for a Keplerian orbit computation can be found in several textbooks e.g. Montenbruck and Gill (2005) and Casotto (1993). Moreover, the Earth orientation matrix has been simplified into the Earth rotation matrix only without considering corrections to the rotation velocity. For the purposes of this analysis, the simplification of the transformation matrix is efficient considering the accuracy of the Keplerian orbits.

This approach has been applied for satellites included in the GPS Blocks IIA, IIR, IIF and GLONASS-M. The attitude analysis has been performed for annual periods during the years 2015 and 2016. The corresponding results of the yaw-angle along with the β angle variations are presented in Figure 3. As it can be seen, the overall number of modelled eclipse manoeuvres of the GPS IIR satellite is almost half in comparison with the GPS IIA, IIF and GLONASS-M satellites. The PWU variations are shown in Figures 4 and 5 along with the differences and their corresponding histograms based on the nominal vs modelled eclipsing yaw-attitude.

In addition, the impact of the PWU correction due to the change of the yaw angle during the eclipse seasons is interpreted in terms of the carrier phase differences for the individual frequencies of the GPS signals. The corresponding phase differences are presented for each GPS frequency (L1, L2 and L5) in Figures 6 and 7. Based on these figures and in particular, the histograms from figures 4 and 5, a zone with significance appears in the range of 45^0 to 90^0 for all of the satellites. In the case of the GPS IIA satellite, the range 90^0 to 180^0 exhibits the highest number of cases. The GLONASS-M satellite exhibits strong disturbances in the overall range. Moreover, the impact of the attitude mis-modelling on the carrier phase discrepancies, as shown in figures 6 and 7, may produce differences more than 10 cm.


GPS IIF, PRN 03

Eclipses: 148
Night turns: 112
Noon turns: 36

GPS IIR, PRN 21

Eclipses: 62
Night turns: 41
Noon turns: 21

GPS IIA, PRN 04

Eclipses: 150
Night turns: 117
Noon turns: 33

Glonass-M, 730

Eclipses: 137
Night turns: 122
Noon turns: 15

Figure 3. Yaw-attitude annual variations for satellites included in the GPS Blocks IIA, IIR IIF and GLONASS-M. The time periods cover the years 2015 in the case of GPS IIA and 2016 for the rest of the satellites. The nominal attitude and the modelled eclipsing attitude are shown along with the β angle variations. The time argument in the X-axis refers to the Modified Julian Day number in days.

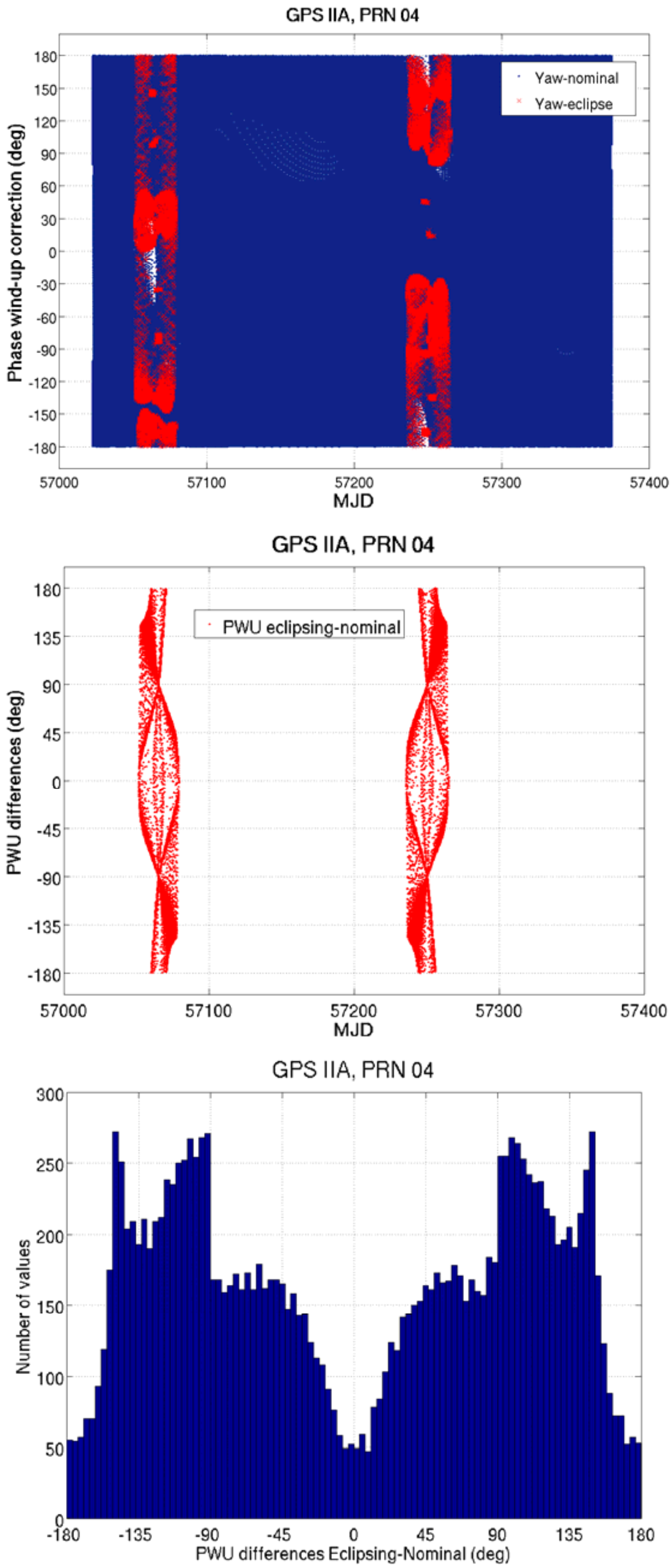


Figure 4. Top: Annual variations of the PWU correction for the IGS station Stromlo located in Australia. Middle and Bottom: The PWU correction differences and their corresponding histograms based on the nominal (blue) and eclipsing yaw-attitude (red) are shown for the GPS satellite PRN04, Block IIA.

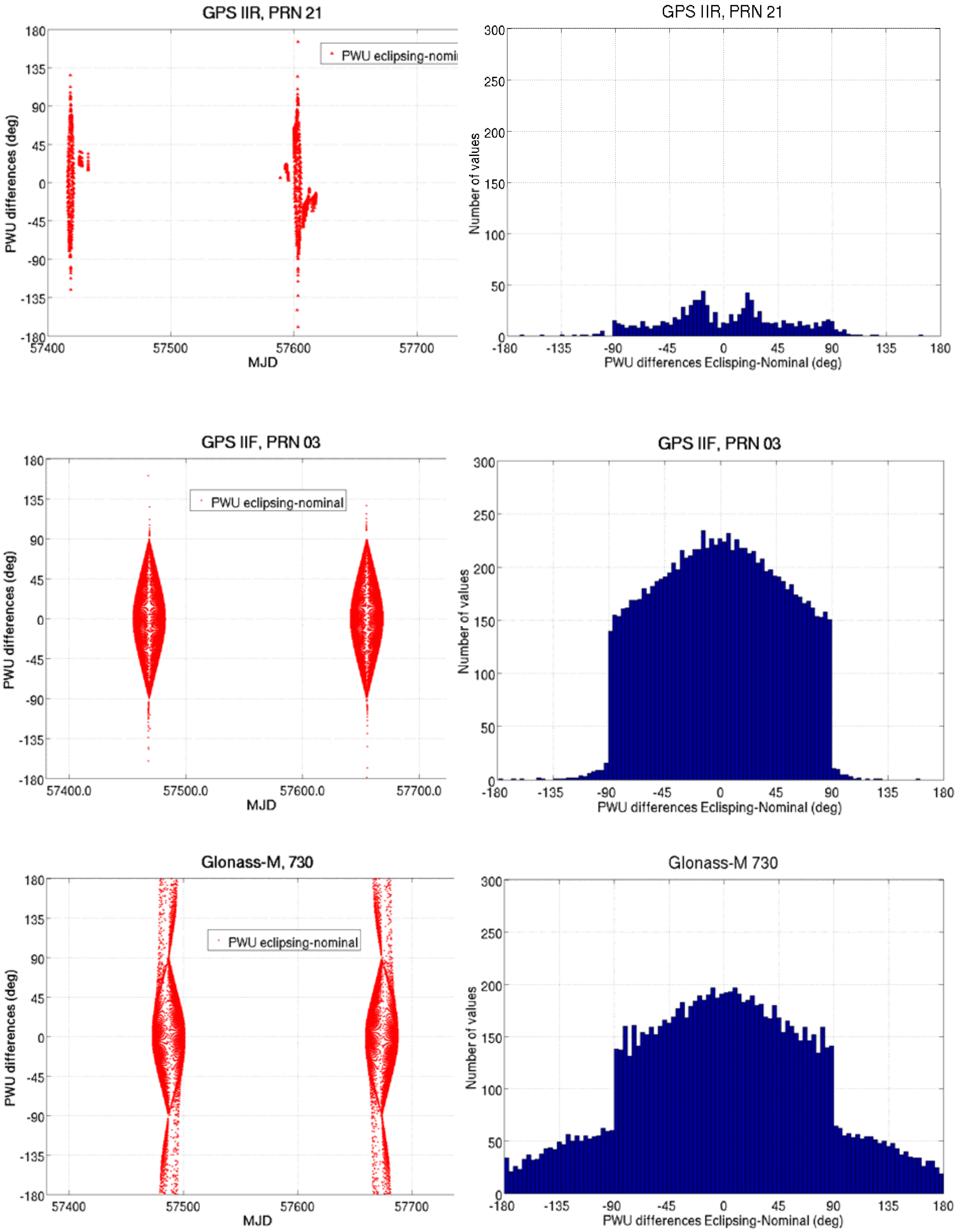


Figure 5. Left: Annual variations of the PWU correction for the IGS station Stromlo located in Australia. Right: The PWU residual histograms based on differences between the nominal (blue) and modelled yaw-attitude during eclipse seasons (red). From top to bottom: GPS Blocks IIR, IIF and GLONASS-M satellites.

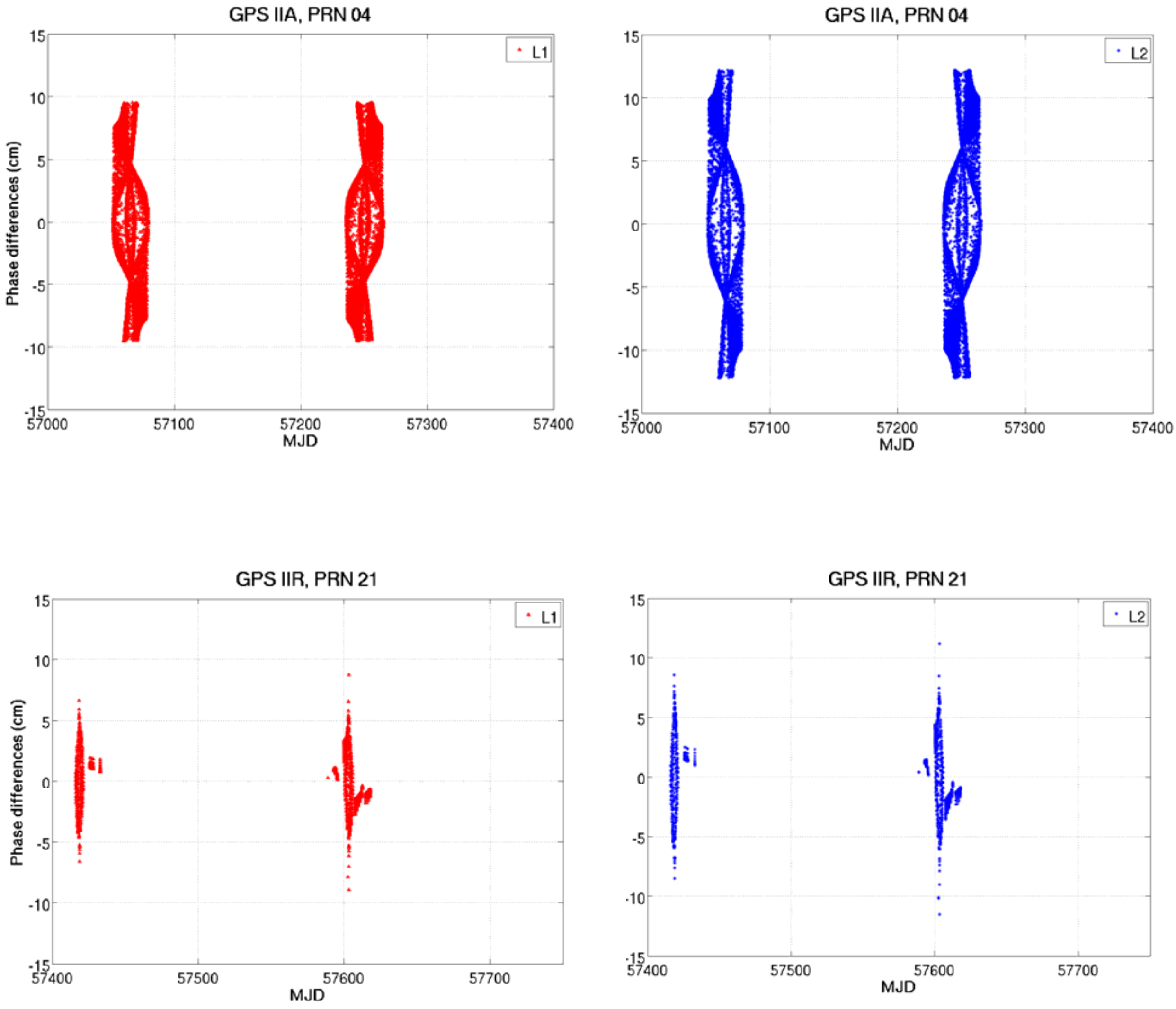


Figure 6. PWU correction differences, based on the yaw-attitude mis-modelling (eclipsing vs nominal), expressed in phase differences in cm for L1 (red) and L2 (blue) for GPS IIA and IIR satellites (top to bottom). The time duration covers two draconic years, 2015 and 2016 respectively, for the IGS station Stromlo located in Australia.

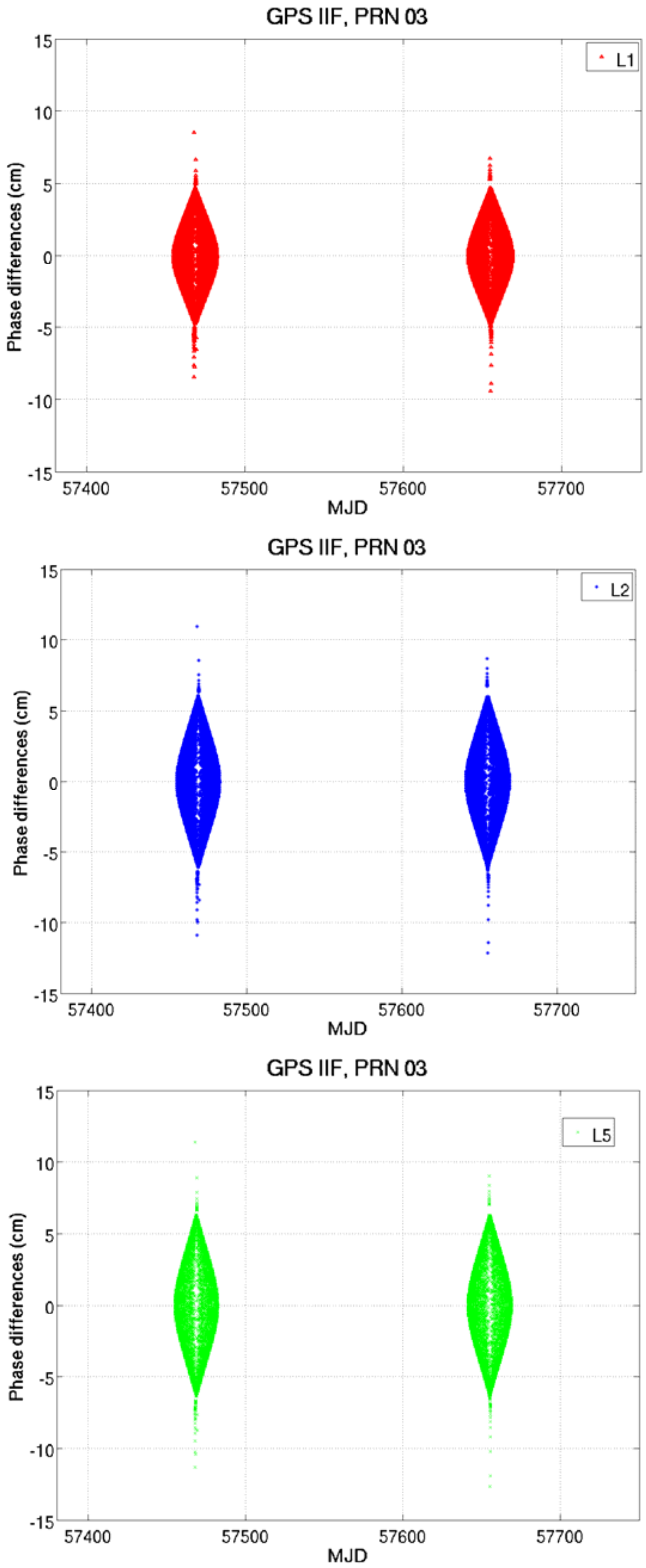


Figure 7. PWU correction differences, based on the yaw-attitude mis-modelling (eclipsing vs nominal), expressed in phase differences in cm for L1 (red), L2 (blue) and L5 (green) for the GPS IIF satellite PRN03. The epochs cover a full draconic year starting from 1 January 2016 for the IGS station Stromlo located in Australia.

The overall number of affected values within an annual period may be of importance in precise analysis of GNSS time series. Thus, prior to such an analysis the affected time periods should be properly modelled. Otherwise, the corresponding observations could be excluded from the analysis if the nominal model is used. However, the last option may worsen the GNSS satellite availability for real time PPP-RTK users.

5. CONCLUSIONS

The present study provides a prediction algorithm for the GNSS yaw-attitude modelling and the PWU correction. In particular, the current study is oriented to the GPS and GLONASS-M satellites. The analysis focuses on the yaw-angle monitoring over annual periods. Such an approach, as applied over a full GNSS draconic year, may reveal the periodical behaviour of the attitude.

In addition, this analysis tool may form a prediction scheme of the GNSS eclipse seasons and the expected impact in the observations through the PWU effect. The monitoring of the differences between nominal and modelled attitude reveal the affected observations and may potentially improve the availability of GNSS satellites during eclipsing season for PPP users. In the case of the GPS IIA satellites the expected impact is critically high since the majority of the epochs have been found to exceed a threshold of $\pm 45^0$ in terms of PWU correction. This number may reach the half of the epochs for the GPS IIF and GLONASS-M satellites while the GPS IIR satellites do not face such critical issues since only a minor part of the computations epochs cross the considered threshold range.

The presented results concentrate to GPS and GLONASS-M constellations. The current modelling must be expanded to the other GNSS constellations of BDS, Galileo, QZSS and GLONASS-K.

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