Receiver Losses when using Quadrature Bandpass Sampling

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ABSTRACT

Use of Quadrature Bandpass Sampling (QBPS) in GNSS receivers gives several advantages over quadrature-sampled superheterodyne architectures. The sampling process has the important secondary function of providing frequency down-conversion (exploiting aliasing) that avoids the use of front-end RF mixers and oscillators. However, the process is not perfect and does incur some losses. This paper examines the use of naïve reconstruction of the QBPS quadrature samples and a simple compensation scheme such as a constant delay of the in-phase channel, and the losses that these methods incur. The paper concludes that for satellite navigation systems, losses incurred due to the use of these simple methods are small enough not to degrade receiver performance.

KEYWORDS: GNSS receivers, sampling, bandpass sampling, quadrature bandpass sampling, image rejection ratio

1. INTRODUCTION

Bandpass sampling [1], where signals with high carrier frequency are sampled at much lower frequencies related to signal bandwidth, has been used in multiband GNSS receiver design [2]. Quadrature bandpass sampling (QBPS) [3] is an extension to this idea which produces sequences similar to those produced by sampling in-phase and quadrature versions of a downconverted signal. Our motivating example is multi-band GNSS [4]. The fact that these sequences are “similar” but not identical to those produced by an analogue front end is what is examined in this paper – what are the penalties paid for this approximation, and can they be tolerated. Following the methods we introduced in [5], we find the answer to these questions is “small” and “yes”.
2. BACKGROUND: QBPS, PROBLEM AND SIMPLE REMEDY

2.1 Sampled Quadrature Downconversion and QBPS

In [5] we considered how a receiver would behave if the more commonly used way of sampling in-phase (I) and quadrature (Q) versions of a received signal, which we refer to as sampled quadrature downconversion as in Figure 1, was replaced with quadrature bandpass sampling (QBPS), as in Figure 2. In QBPS, “downconversion” is a result of exploiting aliasing.

The incoming Radio Frequency (RF) input signal

\[ s(t) = Re\{x(t)e^{j\omega_c t}\} \]

produces, for sampled quadrature downconversion [5]:

\[ i_0(n) = Re\{x(nT_s)e^{j(\omega_c - \omega_{LO})nT_s}\} \]  \hspace{1cm} (2)
\[ q_0(n) = Im\{x(nT_s)e^{j(\omega_c - \omega_{LO})nT_s}\} \]  \hspace{1cm} (3)

and for QBPS [5]:

\[ i_1(n) = Re\{x(nT_s)e^{j\omega_a nT_s}\} \]  \hspace{1cm} (4)
\[ q_1(n) = (-1)^{k+1} I \{ x(nT_s + \Delta t)e^{j\omega_a nT_s} \} \] (5)

where \( \omega_a = \omega_c - 2\pi m/T_s \), \( \Delta t = (2k+1)/4f_c \) and \( \omega_c = 2\pi f_c \). Both \( m \) and \( k \) are integers, with \( m/T_s \) representing the equivalent downconversion frequency achieved through aliasing (usually \( m \) would be chosen to reconstruct at baseband or low-IF), and \( k = 0, 1, \ldots \) selecting the absolute delay between the same sample in the two sequences, with \( k=0 \) giving best results.

The key differences between these approaches, i.e. (2), (3) vs (4), (5), are the intermediate frequency (\( \omega_c-\omega_{LO} \) vs \( \omega_a \)), and the \( \Delta t \) term in (5). For comparison purposes, we can assume \( \omega_c-\omega_{LO} = \omega_a \) so the main concern is the distortion caused by the \( \Delta t \) term. If we perform “naïve reconstruction”, i.e. process \( i_1(n) \) and \( q_1(n) \) of Figure 2 as if they were \( i_0(n) \) and \( q_0(n) \) of Figure 1, then for a given complex frequency within the signal band of interest \( x(t) = e^{j\omega't} \), this results in an image rejection ratio (IRR) [6, 7, 8] of [5]

\[
IRR = 20\log_{10} \left| \cot \frac{\omega' \Delta t}{2} \right| \] (6)

which is worst (lowest) for highest \( \omega' \), i.e. the band edges, or for the whole band [5]

\[
IRR = 20\log_{10} \left( \frac{\sin \left( \frac{\pi B}{2f_c} \right)}{1-\cos \left( \frac{\pi B}{2f_c} \right)} \right) \] (7)

which is constant for a given ratio of bandwidth \( B \) to carrier frequency \( f_c \).

### 2.2 Simple Remedy

A simple remedy to overcome the distortion introduced by QBPS is to apply the same delay \( \Delta t \) to the \( I \)-channel as was experienced in the \( Q \)-channel during sampling, as illustrated in Figure 3 [5]. This has been done using a fractional delay filter [7], although this only works for the special case where \( f_c = r/T_s \), where \( r \) is an integer).

![Figure 3](image)

**Figure 3** Compensating the \( i_1(n) \) channel so processing can proceed as if quadrature downconversion had occurred. Setting \( \Delta t=0 \) produces “naïve reconstruction”

However, because the sampling delay was incurred at the carrier frequency and corrected at the low intermediate frequency, this correction is not perfect and gives an IRR [5] of:

\[
IRR = 20\log_{10} \left| \frac{1+e^{-j\omega_a \Delta t}}{1-e^{-j\omega_a \Delta t}} \right| \] (8)
Note in this case there is a dependence on $\omega_a$, the effective intermediate frequency created by the “downconversion” due to aliasing.

**2.3 Relating IRR to Sampling Rate and Bandwidth**

In [3], we showed that QBPS has the advantage over bandpass sampling (BPS) that it can offer a more comprehensive range of sampling frequencies. This is shown in Figure 4, where the available sampling frequencies ($f_s$) for a given carrier frequency ($f_c$) (each normalised by signal bandwidth $B$) are shaded dark, whereas the extra sampling frequencies available under QBPS are shaded light.

![Figure 4](image-url)  
*Figure 4* Available sampling frequencies versus carrier frequency (normalised by signal bandwidth $B$) for BPS (dark grey) and QBPS (light grey). Note that there are two samples (I and Q) taken at each sampling instant, so the “real” sampling rate is twice that shown. [3]

It is interesting to compare Figure 4 to Figure 5, which is the IRR of (8) plotted for different sampling and carrier frequencies. The “high”, i.e. yellow, values follow the case where the signal is downconverted to zero intermediate frequency, i.e. the special case noted above in [7] where $f_c = r/T_s$, where $r$ is an integer. Those yellow lines (peaks) in Figure 5 follow the centres of alternate wedges in Figure 4. The less obvious troughs in Figure 5 follow the centres of the other alternate wedges in Figure 4.
3. APPLYING QBPS FOR GNSS SIGNALS

In [5], we simply examined the different IRR values of (6), (7), and (8) for GPS L1. Those results are shown in Figure 6 and Figure 7. The many peaks and troughs in Figure 7 are because that figure represents a narrow band very close to 1 for \( f_s/B \) on Figure 5’s y-axis at a point on the x-axis well beyond the extent shown in Figure 5, at \( f_c/B=770 \). The key point to note from Figure 7 is that IRR of at least 72dB, i.e. it is bounded below by (7). This is achievable at any frequency, and that the simple remedy can give better results at some frequencies, and should only be used where it gives an advantage. This IRR is large enough to meet most system requirements so in practice no further complicated reconstruction is necessary.
Figure 6 IRR as calculated by (6) for frequencies across the GPS L1 band [5]

Figure 7 IRR as calculated by (7) (red dashed), and (8) (blue solid) for GPS L1. Note that only a small range of sampling frequencies is shown - because the carrier to bandwidth ratio is 770, there will be 770 peaks in the range B to 2B of sampling frequency [5]

Applying QBPS to a range of GNSS signals gives the minimum IRR from (7) as shown in Table 1. From this table, it can be seen that the large bandwidth signals suffer much greater distortion in the naïve case. It can be seen from Figure 8 that this is also true for the simple remedy. In fact, the basic shape of Figure 8 is much the same as that in Figure 7, normalised by the ratio of the carrier to the bandwidth (as was used for the plot in Figure 5).
Table 1 Achievable IRR for various GNSS signals, in a single frequency receiver.

<table>
<thead>
<tr>
<th>Signal</th>
<th>Carrier Freq. (GHz)</th>
<th>Bandwidth (x 1.023e6 Hz)</th>
<th>IRR (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPS L1</td>
<td>1.57542</td>
<td>2</td>
<td>71.9</td>
</tr>
<tr>
<td>GPS L2C</td>
<td>1.2276</td>
<td>2</td>
<td>69.7</td>
</tr>
<tr>
<td>GPS L5</td>
<td>1.17645</td>
<td>20</td>
<td>49.3</td>
</tr>
<tr>
<td>Galileo E1</td>
<td>1.57542</td>
<td>14</td>
<td>55.0</td>
</tr>
<tr>
<td>Galileo E5</td>
<td>1.191795</td>
<td>50</td>
<td>41.5</td>
</tr>
<tr>
<td>Glonass L1 min</td>
<td>1.5981</td>
<td>1</td>
<td>78.0</td>
</tr>
<tr>
<td>Glonass L1 max</td>
<td>1.6054</td>
<td>1</td>
<td>78.0</td>
</tr>
</tbody>
</table>

Figure 8 The equivalent of Figure 7 (i.e. naïve band red dashed, simple remedy blue solid) except applied to Galileo E5. Note that because the carrier to bandwidth ratio is smaller, a proportionally larger range of frequencies is shown here.

3. CONCLUSIONS

It can be seen that quadrature bandpass sampling (QBPS) can be readily applied to satellite navigation signals, and that even without correction for the fact that sampling is not “perfectly in quadrature”, the distortion is quite small, especially where the ratio of the carrier frequency to the bandwidth is high. Where wider bandwidth signals are used, image rejection ratio (IRR) can drop to as low as 41dB for Galileo E5. Use of the simple remedy of delaying the samples in the in-phase sequence can give much better results, if the sampling frequency is selected appropriately. Depending on the application, however, the 41dB IRR may be considered acceptable.
REFERENCES


