UNSW AUSTRALIA

Integrated Techniques for Interference Source Localisation in the GNSS band

Never Stand Still | Faculty of Engineering | Australian Centre for Space Engineering Research (ACSER)

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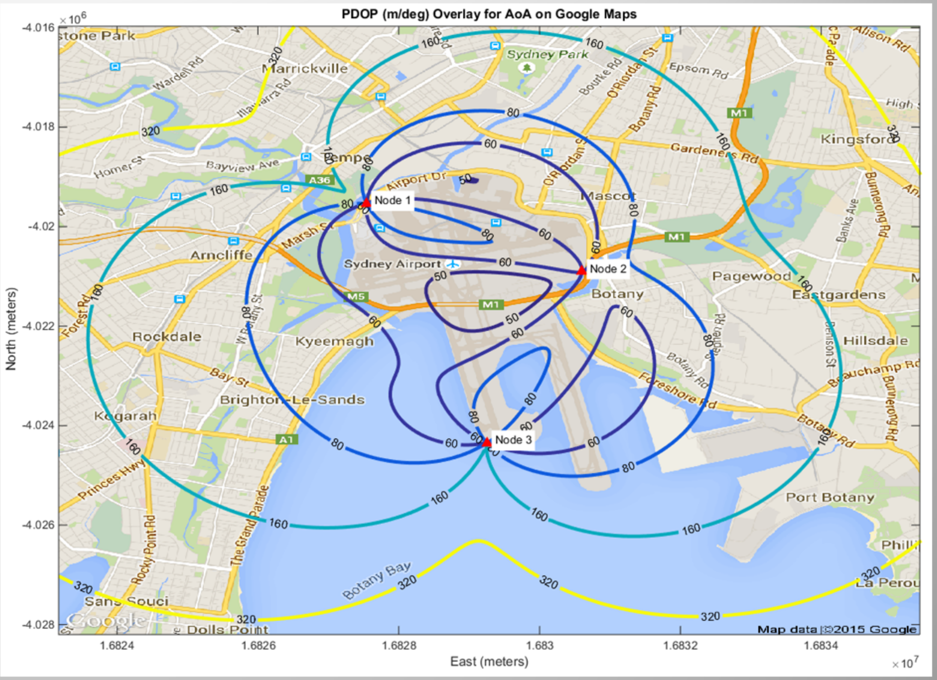
SNAP Lab

Introduction

- GNSS signals are inherently weak
- Spurious transmissions and intentional jammers in the GNSS band threatens safety critical applications that depends on GNSS
- A network of sensors tuned to the GNSS band can be used to detect the angle of arrival (AOA) and time difference of arrival (TDOA) of the jammer.

Introduction



- AOA: Uses phased antenna arrays DSP
- TDOA: Uses cross-correlation method DSP
- Geo-localisation of jammer
  - AOA: Intersection of lines
  - TDOA: Intersection of hyperbolas
  - Can we combine AOA and TDOA for Geo-localisation?

Jammer Characteristics

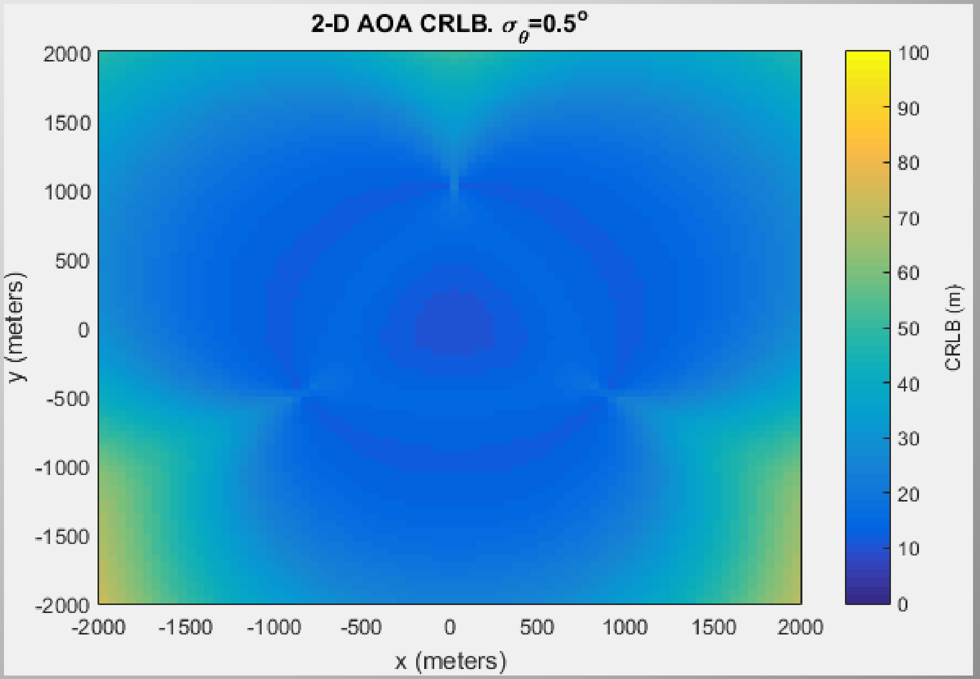
- Narrowband
  - Strong jammer signal strength will affect receiver performance
  - Can be detected using AOA
- Wideband
  - Weak jammer signal strength is sufficient to affect receiver performance
  - Can be detected using TDOA and AOA

Cramer Rao Bound: AOA

AOA Measurement Covariance $\Sigma_\theta \in \mathbb{R}^{N\times N}$

Jacobian $J_H \leftarrow \begin{bmatrix} -\frac{y-y_1}{r_1^2} & \frac{x-x_1}{r_1^2} \\ \vdots & \vdots \\ -\frac{y-y_N}{r_N^2} & \frac{x-x_1}{r_N^2} \end{bmatrix}$

CRB $\left(J_H^T \Sigma_\theta^{-1} J_H\right)^{-1}$



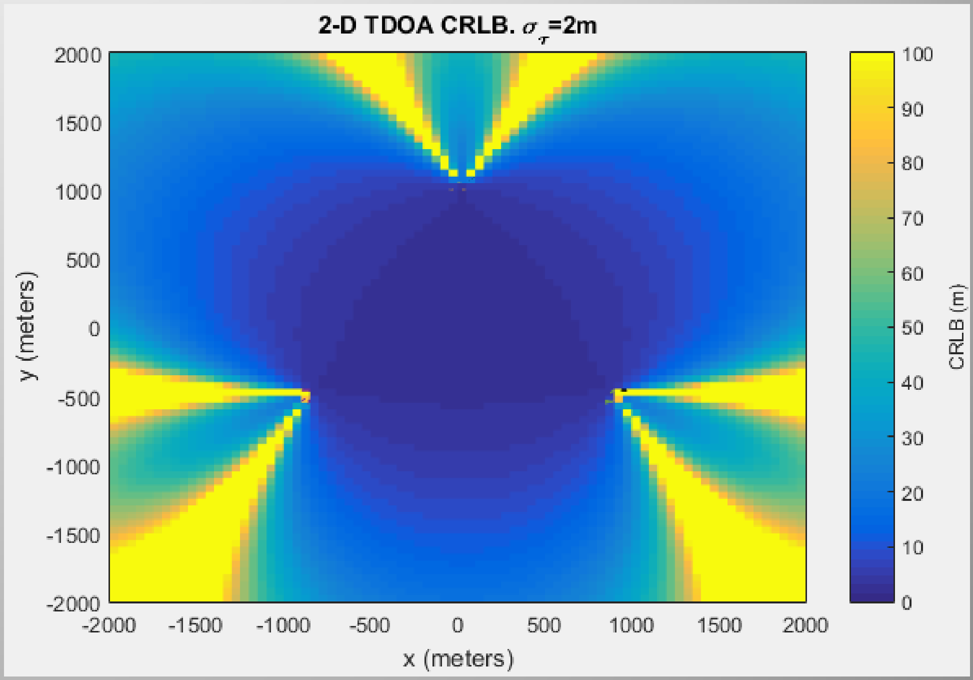
- Most of the errors are within 10-40m
- Errors behave smoothly outside the convex area

Cramer Rao Bound: TDOA

TDOA Measurement Covariance: $\Sigma_\tau \in \mathbb{R}^{N-1\times N-1}$

$$J_H \leftarrow \begin{bmatrix} \frac{x-x_2}{r_2} - \frac{x-x_1}{r_1} & \frac{y-y_2}{r_2} - \frac{y-y_1}{r_1} \\ \vdots & \vdots \\ \frac{x-x_N}{r_N} - \frac{x-x_1}{r_1} & \frac{y-y_N}{r_N} - \frac{y-y_1}{r_1} \end{bmatrix}$$

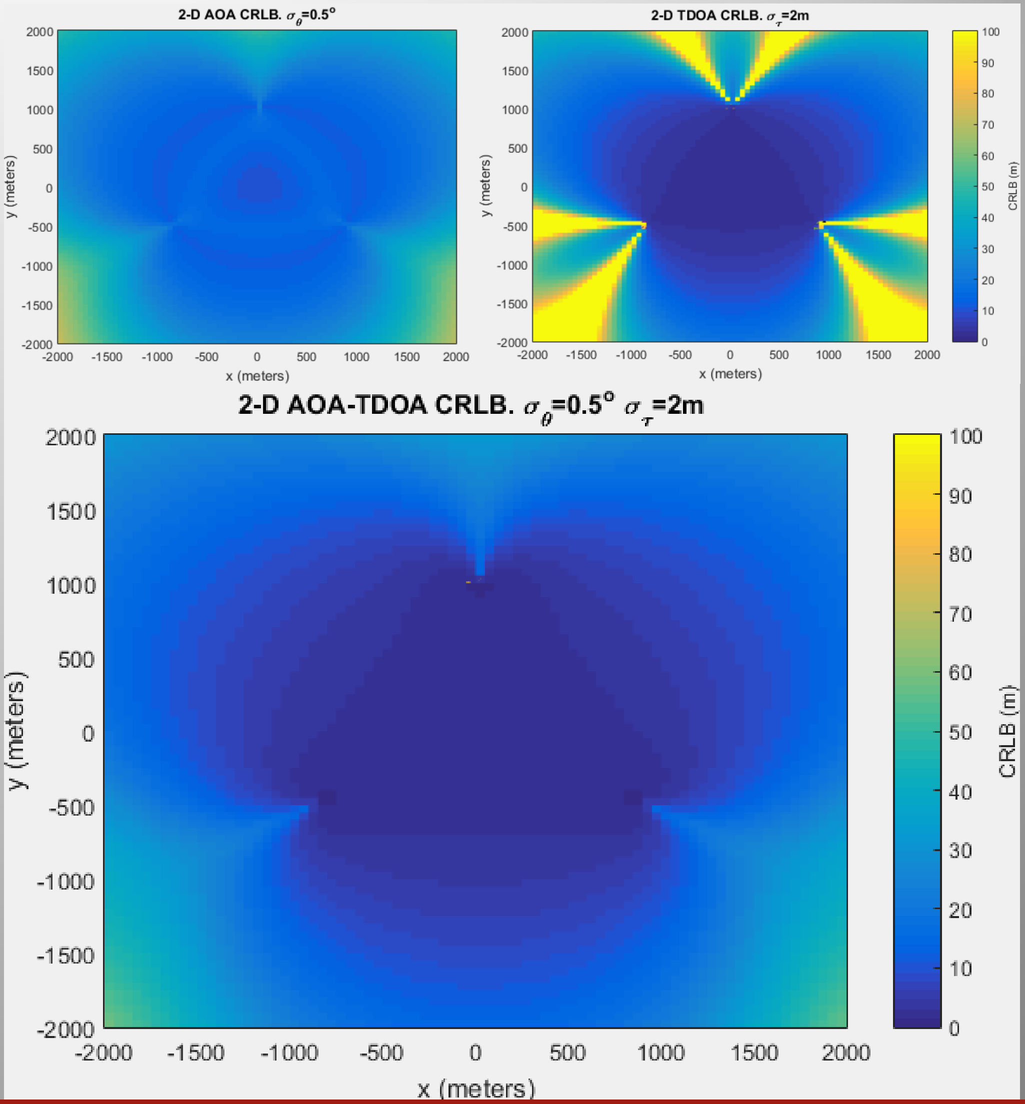
CRB: $(J_H^T \Sigma_\tau^{-1} J_H)^{-1}$



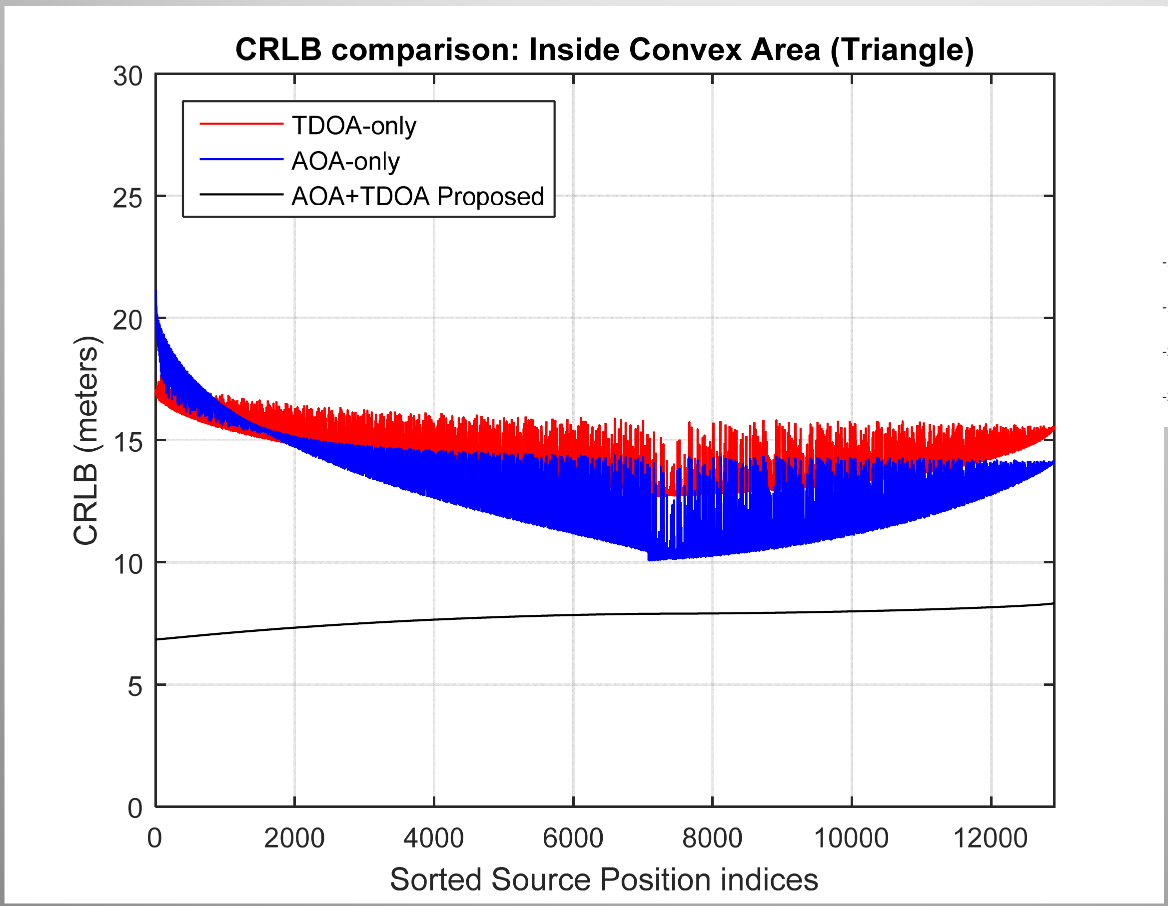
- Most of the errors are within 5-40m
- Errors behave erratically due to rank deficiency beyond the convex area bounded by the 3 nodes

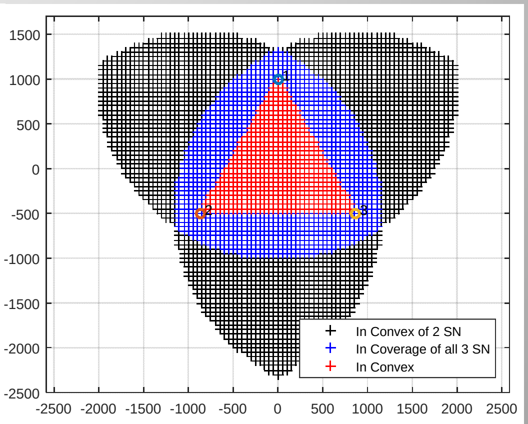
CRB for AOA + TDOA Integration

- Most of the errors are within 2-30m
- Rank deficient regions significantly improved
- Lowest CRLB achieved at all points



Fair comparison between independent localisation and integrated localisation



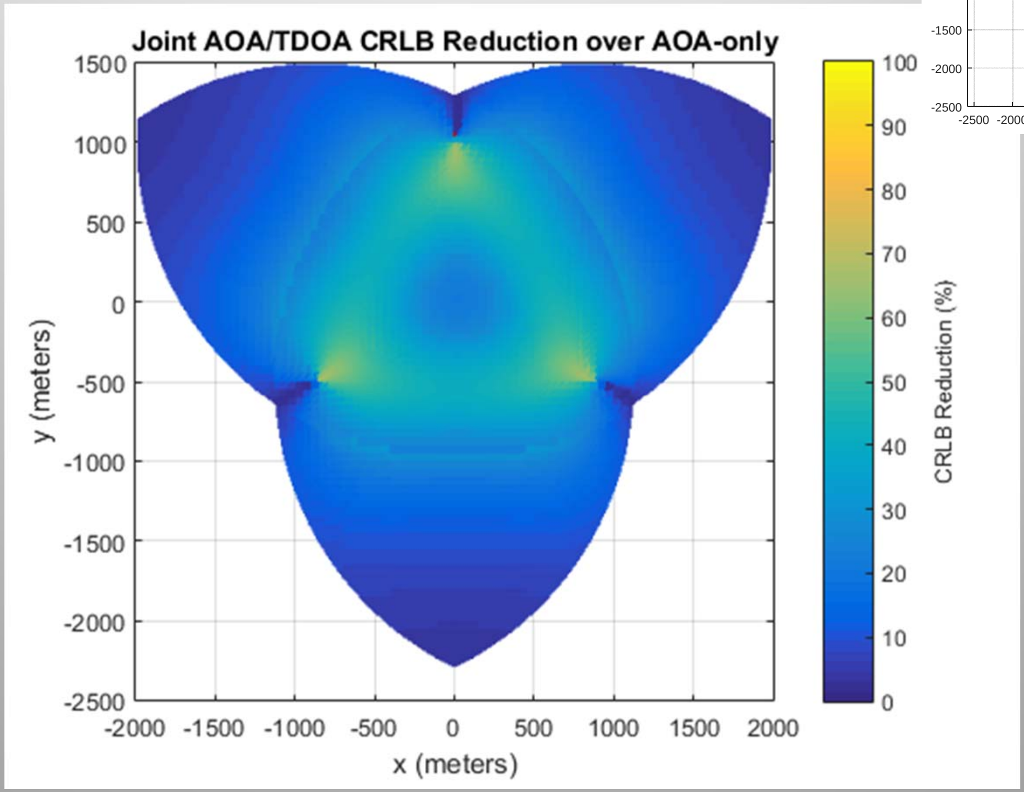


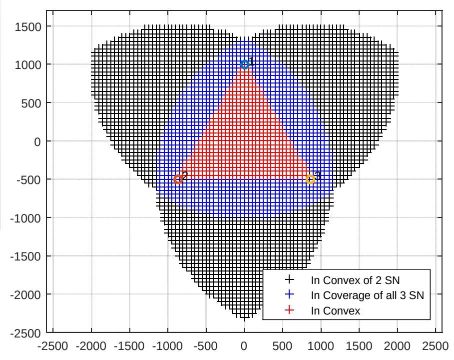
AOA STD: 0.5 degrees

TDOA STD: 11m

Improvement over AOA-only

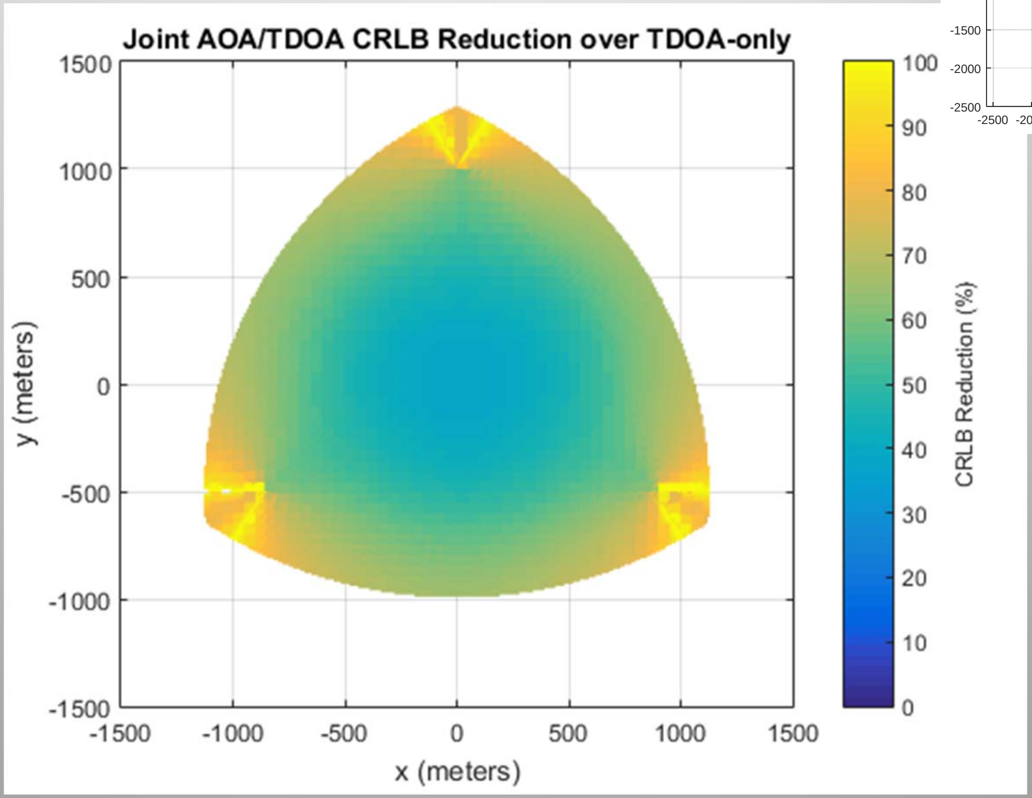
- Improvement measured in percentage (%)

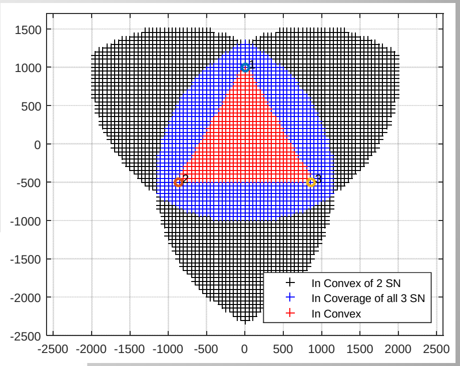




Improvement over TDOA-only

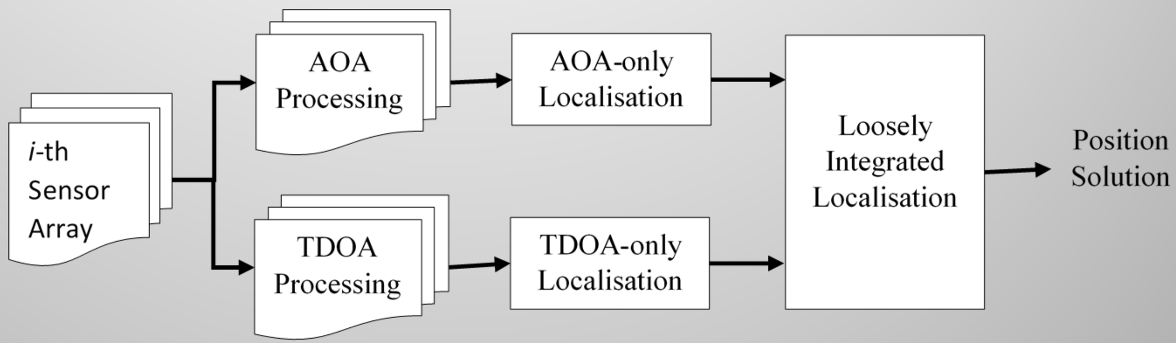
- Improvement measured in percentage (%)



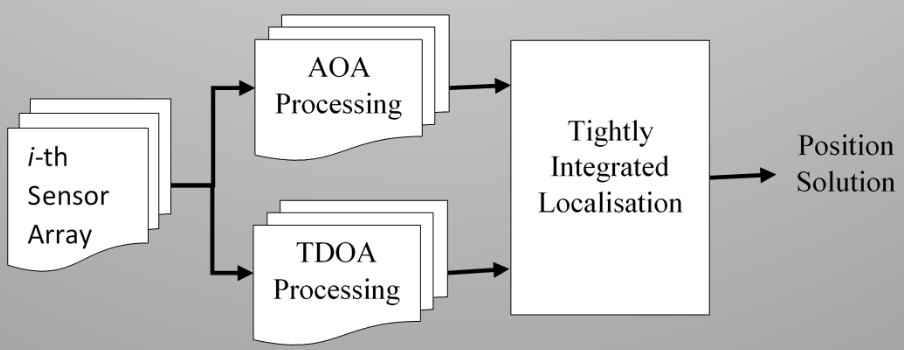


AOA + TDOA Fusion Architectures

Loose Integration



Tight Integration



Loose Integration Algorithm

Input: AOA measurements $\theta_i, \forall i \in \{1, \ldots, N\}$
TDOA measurements $\hat{\tau}_{i1}, \forall i \in \{2, \ldots, N\}$
Sensor Node Positions $x_i, y_i, \forall i \in \{1, \ldots, N\}$
Source Guesstimate Position x_0, y_0
AOA Noise Covariance Matrix $\Sigma_\theta \in \mathbb{R}^{N \times N}$
TDOA Noise Covariance Matrix $\Sigma_\tau \in \mathbb{R}^{N-1 \times N-1}$

Output: $\hat{x}, \hat{y}$ Estimated Emitter Position

Initialise $\hat{x}, \hat{y} \leftarrow x_0, y_0$

Compute TDOA-only solution with arguments: $\hat{\tau}_{i1}, x_i, y_i, \Sigma_\tau$
Output stored as x_T, y_T

Compute AOA-only solution with arguments: $\theta_i, x_i, y_i, \Sigma_\theta$
Output stored as x_A, y_A

Compute Position Error Covariance Matrix for x_T, y_T

$$r_i \leftarrow \left\| \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} - \begin{bmatrix} x_i \\ y_i \end{bmatrix} \right\|$$

$$J_T \leftarrow \begin{bmatrix} \frac{x_0 - x_2}{r_2} - \frac{x_0 - x_1}{r_1} & \frac{y_0 - y_2}{r_2} - \frac{y_0 - y_1}{r_1} \\ \vdots & \vdots \\ \frac{x_0 - x_N}{r_N} - \frac{x_0 - x_1}{r_1} & \frac{y_0 - y_N}{r_N} - \frac{y_0 - y_1}{r_1} \end{bmatrix}$$

$$\Sigma_T \leftarrow (J_T^T \Sigma_\tau^{-1} J_T)^{-1}$$

Compute Position Error Covariance Matrix for x_A, y_A

$$J_A \leftarrow \begin{bmatrix} -\frac{y_0 - y_1}{r_1^2} & \frac{x_0 - x_1}{r_1^2} \\ \vdots & \vdots \\ -\frac{y_0 - y_N}{r_N^2} & \frac{x_0 - x_1}{r_N^2} \end{bmatrix}$$

$$\Sigma_A \leftarrow \left(J_A^T \Sigma_\theta^{-1} J_A\right)^{-1}$$

Perform Loose Integration

$$\Sigma = \begin{bmatrix} \Sigma_A & 0_{2\times2} \\ 0_{2\times2} & \Sigma_T \end{bmatrix}$$

$$J_H \leftarrow \begin{bmatrix} I_{2\times2} \\ I_{2\times2} \end{bmatrix}$$

$$\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} \leftarrow (J_H^T \Sigma^{-1} J_H)^{-1} J_H^T \Sigma^{-1} \begin{bmatrix} x_A \\ y_A \\ x_T \\ y_T \end{bmatrix}$$

Loose Integration Algorithm

Input: AOA measurements $\theta_i, \forall i \in \{1, \dots, N\}$

TDOA measurements $\hat{\tau}_{i1}, \forall i \in \{2, \dots, N\}$

Sensor Node Positions $x_i, y_i, \forall i \in \{1, \dots, N\}$

Source Guesstimate Position x_0, y_0

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TDOA Noise Covariance Matrix $\Sigma_\tau \in \mathbb{R}^{N-1 \times N-1}$

Output: $\hat{x}, \hat{y}$ Estimated Emitter Position

Initialise $\hat{x}, \hat{y} \leftarrow x_0, y_0$

Compute TDOA-only solution with arguments: $\hat{\tau}_{i1}, x_i, y_i, \Sigma_\tau$

Output stored as x_T, y_T

Compute AOA-only solution with arguments: $\theta_i, x_i, y_i, \Sigma_\theta$

Output stored as x_A, y_A

Compute Position Error Covariance Matrix for x_T, y_T

$$r_i \leftarrow \left\| \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} - \begin{bmatrix} x_i \\ y_i \end{bmatrix} \right\|$$

$$J_T \leftarrow \begin{bmatrix} \frac{x_0 - x_2}{r_2} - \frac{x_0 - x_1}{r_1} & \frac{y_0 - y_2}{r_2} - \frac{y_0 - y_1}{r_1} \\ \vdots & \vdots \\ \frac{x_0 - x_N}{r_N} - \frac{x_0 - x_1}{r_1} & \frac{y_0 - y_N}{r_N} - \frac{y_0 - y_1}{r_1} \end{bmatrix}$$

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Compute Position Error Covariance Matrix for x_A, y_A

$$J_A \leftarrow \begin{bmatrix} -\frac{y_0 - y_1}{r_1^2} & \frac{x_0 - x_1}{r_1^2} \\ \vdots & \vdots \\ -\frac{y_0 - y_N}{r_N^2} & \frac{x_0 - x_1}{r_N^2} \end{bmatrix}$$

$$\Sigma_A \leftarrow (J_A^T \Sigma_\theta^{-1} J_A)^{-1}$$

Perform Loose Integration

$$\Sigma = \begin{bmatrix} \Sigma_A & 0_{2\times2} \\ 0_{2\times2} & \Sigma_T \end{bmatrix}$$

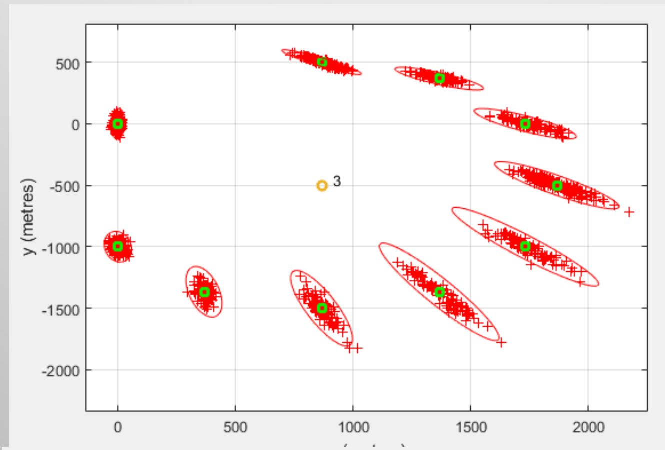
$$J_H \leftarrow \begin{bmatrix} I_{2\times2} \\ I_{2\times2} \end{bmatrix}$$

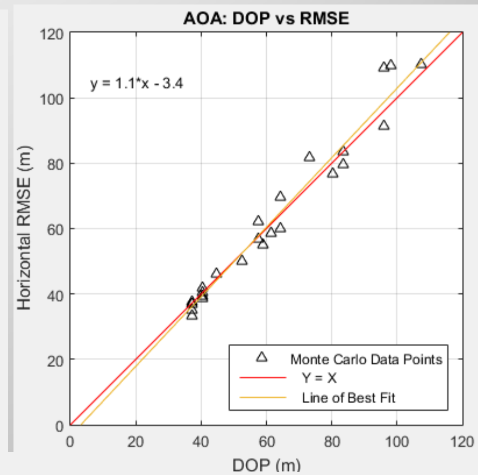
$$\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} \leftarrow (J_H^T \Sigma^{-1} J_H)^{-1} J_H^T \Sigma^{-1} \begin{bmatrix} x_A \\ y_A \\ x_T \\ y_T \end{bmatrix}$$

- The key to loose integration is the computation of an accurate Position Error Covariance Matrix for AOA and TDOA systems.
- requires an approximate position to be provided
- provides a “weighing” mechanism

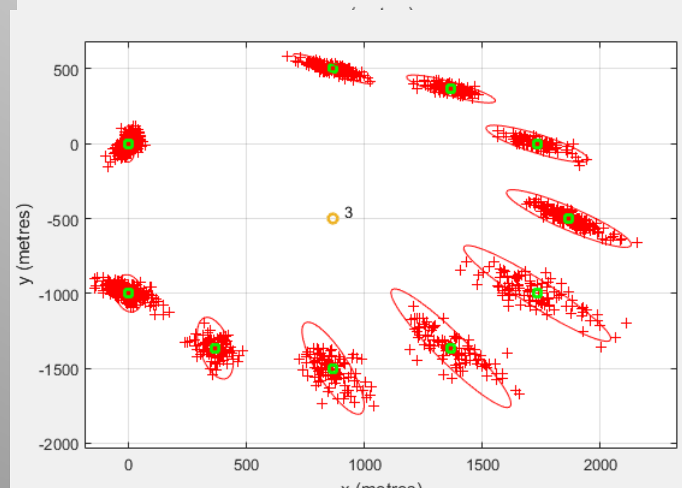
Effect of incorrect weighing matrix

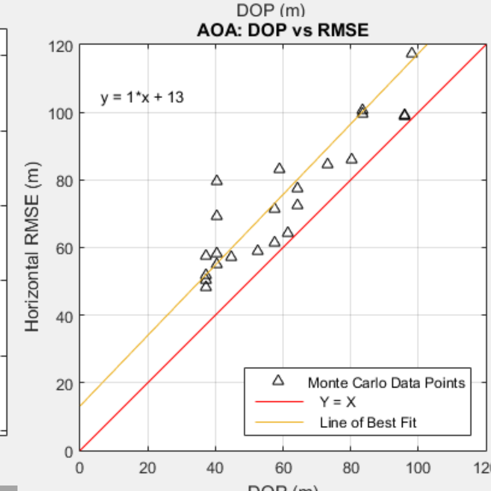
Correct Weighing





Incorrect Weighing





Tight Integration Algorithm

Input: AOA measurements $\theta_i, \forall i \in \{1, \dots, N\}$

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TDOA Noise Covariance Matrix $\Sigma_\tau \in \mathbb{R}^{N-1 \times N-1}$

Output: $\hat{x}, \hat{y}$ Estimated Emitter Position

Iterate:

$$x, y \leftarrow \hat{x}, \hat{y}$$

$$r_i \leftarrow \left\| \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} x_i \\ y_i \end{bmatrix} \right\|$$

$$J_H \leftarrow \begin{bmatrix} -\frac{y - y_1}{r_1^2} & \frac{x - x_1}{r_1^2} \\ \vdots & \vdots \\ -\frac{y - y_N}{r_N^2} & \frac{x - x_1}{r_N^2} \\ \frac{x - x_2}{r_2} - \frac{x - x_1}{r_1} & \frac{y - y_2}{r_2} - \frac{y - y_1}{r_1} \\ \vdots & \vdots \\ \frac{x - x_N}{r_N} - \frac{x - x_1}{r_1} & \frac{y - y_N}{r_N} - \frac{y - y_1}{r_1} \end{bmatrix}$$

$$\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \leftarrow (J_H^T \Sigma^{-1} J_H)^{-1} J_H^T \Sigma^{-1} \begin{bmatrix} \Delta\theta_1 \\ \vdots \\ \Delta\theta_N \\ \tau_{21} \\ \vdots \\ \tau_{N1} \end{bmatrix}$$

$$\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} \leftarrow \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix}$$

End

Conclusion

- AOA and TDOA Integration provides superior performance under all circumstances
- Existing attempts to combine AOA and TDOA has been suboptimal due to incorrect "weighing" and/or use of a Loose Integration Architecture
- 2 architectures has been proposed that can be adapted to various existing platforms
- Proposed algorithms of both architectures approaches the Cramer Rao Lower Bound

Acknowledgement

- GPSat Systems Australia
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Questions?

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